Bending-Active Segmented Shells

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Abstract

This paper presents a new structural system that combines the benefits of segmented shells and active bending. Initially planar plate elements are bent only by support displacements and then connected to form shell structures. This advantageous procedure allows building continuous freeform surfaces from single-curved, developable elements with a very simple and economic construction process, while achieving the structural efficiency of a shell. An innovative form finding approach is presented: an explicit variation of the bending stiffness along a segment, allows manipulating its spatial, actively bent shape to closely approximate a designed target shell geometry. The accuracy of this process and the structural behaviour of the shell under external loads and in different scales is discussed.

Keywords: bending-active, segmented shells, freeform shells, bending stiffness, scale, simulation, Finite-Element Analysis

1. Introduction

A common trend in architecture is the design of freeform surfaces for roofs, facade systems or temporary pavilions, etc. Shells are the most promising structural systems for these designs, as they allow for an efficient use of building materials by combining structure and form.

Frequently, double-curved freeform shells are discretised through a reticulated grid system with linear elements and erected on a scaffold. Alternatively, closed shell surfaces can be built using extensive formwork. Both construction methods can become cost and labour intensive.

Segmented shells with e.g. planar elements (Fig. 1) are a step towards a simpler construction method, which still achieves the structural efficiency of a continuous shell surface (Li and Knippers [5]).

Bending-active structures are systems that show spatial and curved shapes obtained from initially straight or planar elements (Lienhard [6]). Their complex geometry is derived from large elastic bending deformations. Lienhard *et al*. [7] give an overview on the use of actively bent elements as structural systems.

Figure 1: LAGA Exhibition Hall (Brütting 2014), Multihalle Mannheim (Brütting 2016), Plydome (Fuller [10])

Furthermore, active bending can be applied to construct shell-like freeform structures such as the prominent Multihalle in Mannheim (Fig. 1). Here a net of slender rods is topologically arranged in a flat state and simply bent into a global double-curved shape on site. Buckminster Fuller [10] takes a step further towards closed surfaces by building polyhedral domes through the connection of locally bent plywood plates at their corners (Fig. 1). Similarly, La Magna *et al.* [4] present freeform shapes achieved through the topological arrangement and interweaving of flexible sheets.

2. Aim of the Research

By combining the benefits of segmented shells and active bending, the approach of Bending-Active Segmented Shells is to approximate global double-curved freeform shells through single curved plates (strips). The plates are bent by only moving the endpoints of initially flat elements into their final position (Fig. 2), achieving a simple, moldless construction process. Therefore, this paper introduces analytical solutions for the derivation of exact stiffness gradients along bent elements to closely approximate designed freeform shapes with bending-active elements. The connection of multiple strips to a double curved structure allows establishing a global load transfer by membrane action. This is highly beneficial because the strips must be thin to enable the initial bending formation.
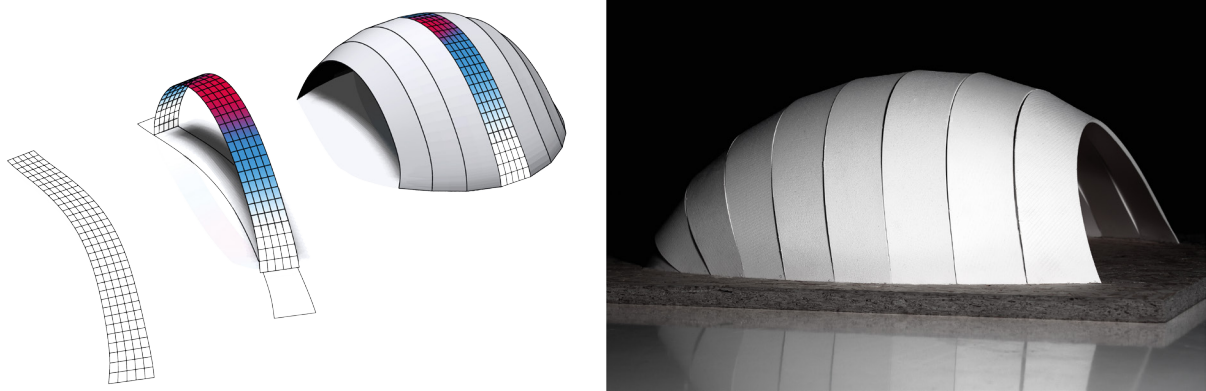

Figure 2: Left: Construction process of a Bending-Active Segmented Shell.
Right: Prototype with variable cross-section thickness and a span of 70 cm

3. Methodology

3.1. Segmentation

The target surface as the initial design of a shell structure is a double-curved surface. It can be easily designed and changed in a 3D-CAD environment (here: *Rhinoceros®*). The surface must be subdivided into single-curved, developable elements which can be unrolled and produced from flat sheet material (Fig. 3). This is a common problem in architectural geometry (Pottmann *et al.* [13], Anastas *et al.* [2]).

Liu *et al.* [9] create developable strips from a recursive planarisation and subdivision of a quad mesh. This approach is followed by using the particle-spring system Kangaroo, developed by Piker [12]. A fine, planarised quad mesh, approximates the target surface. To achieve planarity, the vertices move in space, yet the mesh should remain close to the target geometry to approximate the designed shape as good as possible: trade-offs have to be made or the shell design adjusted. The vertices of the planarised mesh then are used as interpolation points for curves representing the edges of developable strips.
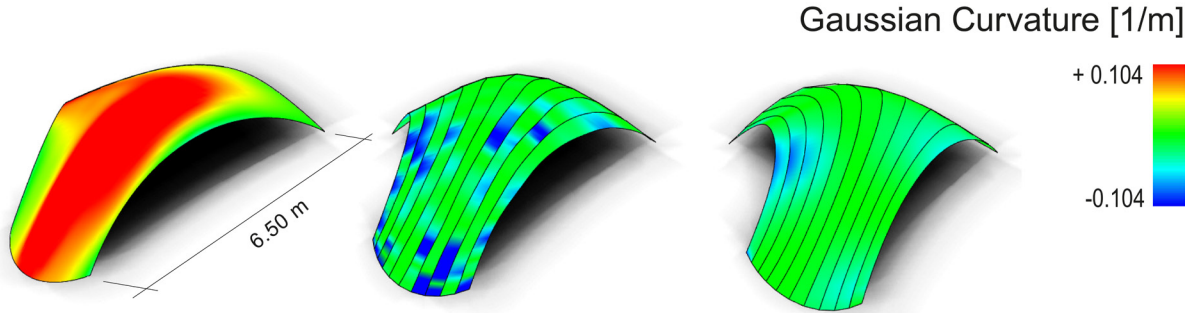
Figure 3: Progress of the particle-spring system Kangaroo to create single-curved strips through mesh planarization

3.2. Free Form Generation

3.2.1 The Elastica

The basic Elastica can be seen as the post-buckling curve of a beam between two pinned supports with an applied normal force. Through a variation of the actuation force differently shaped Elastica curves are achieved. However, the design freedom between two pinned supports is very limited using an isotropic material and a constant cross section – the curves are e.g. always symmetric.

For the slender elements, the Euler-Bernoulli Beam Theory relates the curvature of a bent profile to the applied bending moment:
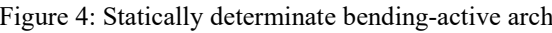
$$\kappa = \frac{M}{EI}$$

(1)

3.2.2 Stiffness Gradients

If producing bending-active elements with more variability in shape is the aim, one approach is the introduction of tension elements (Alpermann and Gengnagel [1]). Alternatively, one can adapt the bending stiffness along the axis of the element to manipulate the curved shape. Nicholas and Tamke [11] showed this approach on an experimental basis. Bechert *et al.* [3] present the forming of shell components which achieve a designed curvature by bending custom laminated plywood into closed loops under an approximately constant bending moment.

For statically determinate systems, it is possible to calculate the internal forces at any point of a structure from the system's geometry and the applied loads alone, independently of stiffnesses of individual members. In some cases, this correlation is reversible and one can design a curved, statically determinate system and derive the necessary bending stiffness for a given loadcase to achieve exactly the designed shape after a large elastic bending deformation.

$$EI = \frac{M}{\kappa} = \frac{Ph}{\kappa}$$

(2)

For a statically determinate bending-active arch with one pinned and one roller support (Fig. 4) the bending moment *M* at any point *i* along the curve can be calculated directly from the actuation force. This allows deriving an exact stiffness gradient to bend the member into the designed geometry.

Figure 4: Statically determinate bending-active arch

For simplicity, here the cross section is considered as constant throughout the element. Eq. (2) can be formulated excluding P and with an upper bound representing a maximal possible Young´s modulus E_{max} of e.g. a given anisotropic material.

$$E_i = \frac{h_i}{\kappa_i} \cdot \frac{1}{\alpha} \cdot E_{max} \qquad \alpha = \max(\frac{h_i}{\kappa_i}) \tag{3}$$

Apart from the explicit change of the Young´s modulus, manipulating the area moment of inertia is an alternative. The equations for a distribution of a variable thickness or the complete bending stiffness EI can be derived equivalently from (2).

Fig. 5 shows necessary stiffness distributions to achieve designed shapes through active bending. Additionally, lower bounds of the gradient and singular points e.g. at the pinned supports are considered.

Figure 5: Stiffness gradients for different bending-active arches

For bending-active shell segments the presented method is expanded to 2D plate elements by evaluating the remaining principle curvature and the width of the generated developable strips.

3.2.3 Materialisation

Flexible, yet high-strain resistant materials are suitable for bending-active constructions. Wood and fibre-reinforced polymers are ideal for this purpose. As previously presented, the bent shape of a bending-active element can be influenced by changing the bending stiffness *EI* along the segment. Varying the effective Young's modulus of the cross section and the cross-section thickness allows generating this gradient. The anisotropy and directional properties of wood and FRPs can be used to customise bending-active segments with the required stiffness gradients e.g. through additive manufacturing techniques. Furthermore, it is possible to increase the global structural performance of the shell through this custom stiffness placement.

An alternative are elements with changing thickness only, fabricated from an isotropic material by subtractive manufacturing. This could be shown through a small-scale functioning prototype (Fig. 2).

3.3. Simulation and Analysis Tool

In this work, Finite Element Simulations are carried out taking into account a transversely isotropic material behaviour with variable stiffness and local orientation, in order to simulate the large deformations mechanically correct in an iterative, fully geometric nonlinear analysis. This is made possible through the translation of CAD geometry into the syntax of the commercial software *SOFISTIK®*.

In the case of bending-active segmented shells, multiple bent strips are then coupled to a closed surface by joining their adjacent edges. This process is simulated via cable elements contracted to approximately zero length through negative strain. The resulting segmented shell structure can be evaluated for external loads.

Through custom-developed tools, it is possible to re-import and visualise the results of the FE-Analysis in the parametric design environment Grasshopper®. This allows analysing and manually improving the global shell geometry or the segmentation in areas of unsatisfactory structural performance.
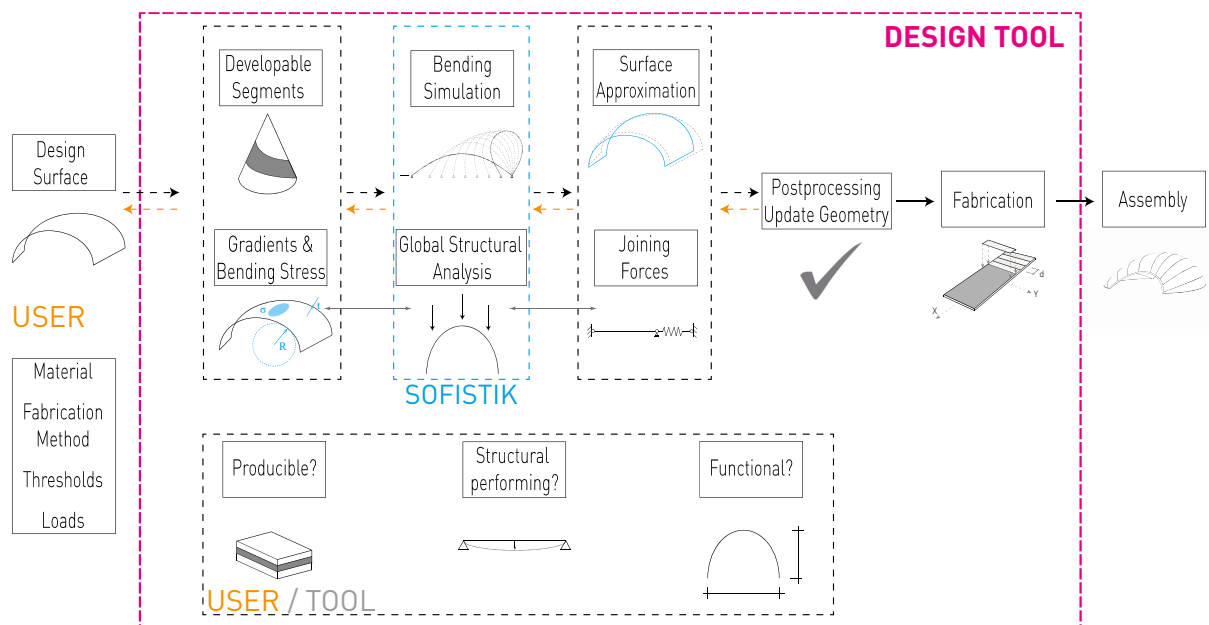

Figure 6: Design process for bending-active segmented shells combined in an integrative design tool

4. Results

4.1. Design Surface Approximation and Joining

The method of adapted stiffness is derived in a planar system and extended to surface elements. However, most segments of freeform shells show distorted unrolled shapes. This influences the quality of approximating the target shell geometries through bending (Fig. 6). Straight strips (A, B) can closely fit the target design. For undulating strip geometries (C, D) spatial bending directions are predominant.
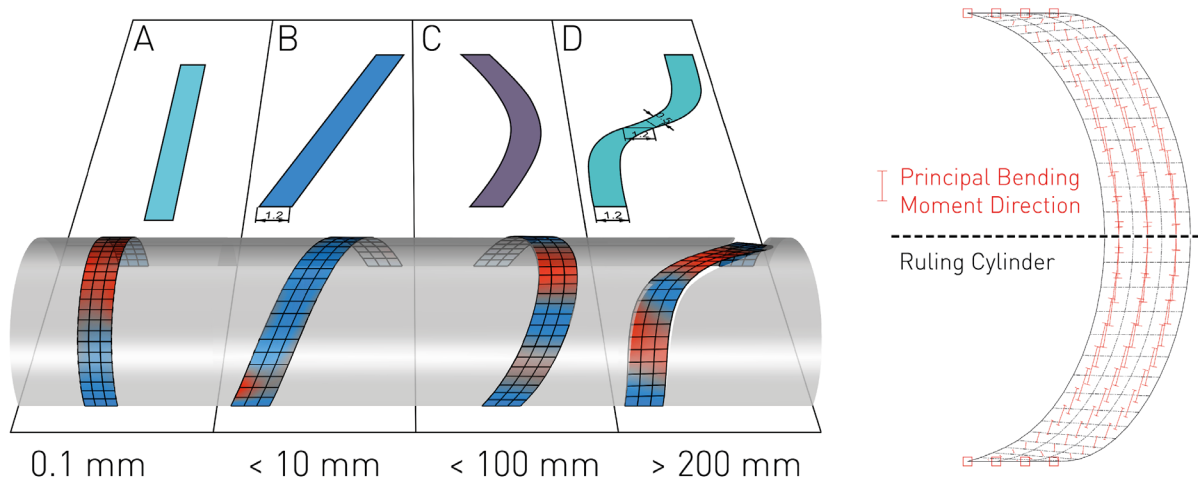

Figure 7: Left: Deviation of different strip geometries from a cylindrical surface (R = 3.00 m).
Right: Top view - orientation of the principle bending moment after the bending simulation (C)

It can be concluded that the developed method to approximate freeform design shapes with bending active elements holds for planar systems. In the case of spatial plate systems with strips of complex geometries, the actuation directions are often non-parallel and do not line up with the strips. This causes principal bending moments that are not exactly aligned orthogonally to the rulings of the surface (Fig. 7, right). Thus the bent shape can only approximate the stipulated design.

Deviations from the target geometry can be minimized by choosing appropriate shell and strip geometries. For further steps in the design process, one would use the simulated geometry as new reference geometry.

For the examined shell geometries, adjacent bent strips with adapted stiffness show only small deviations compared to reference elements with overall constant bending stiffness (e.g. isotropic material). This further reduces the forces necessary to join segments to a closed surface (Fig. 8).
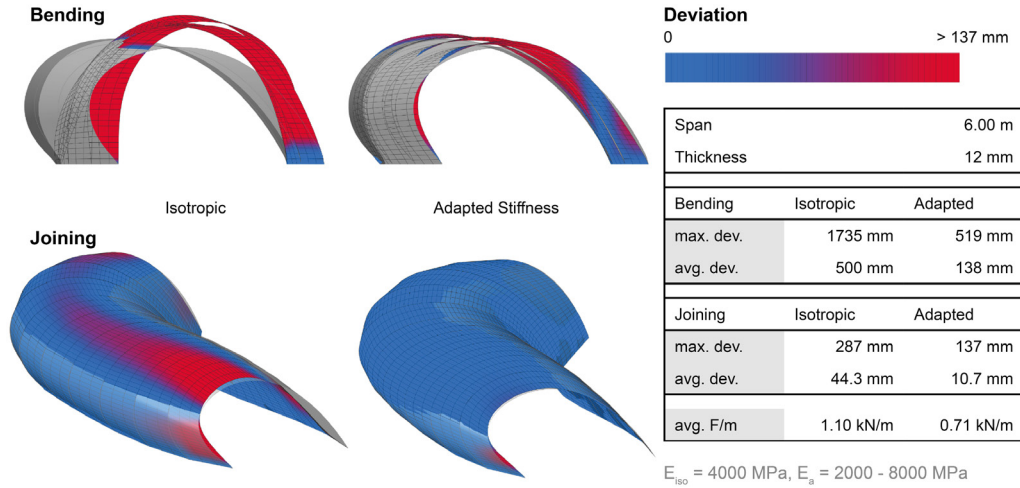
Bending		
Isotropic	Adapted Stiffness	

Joining		

Deviation		
0		> 137 mm

Span		6.00 m
Thickness		12 mm
Bending	Isotropic	Adapted
max. dev.	1735 mm	519 mm
avg. dev.	500 mm	138 mm
Joining	Isotropic	Adapted
max. dev.	287 mm	137 mm
avg. dev.	44.3 mm	10.7 mm
avg. F/m	1.10 kN/m	0.71 kN/m

E_{iso} = 4000 MPa, E_a = 2000 - 8000 MPa

Figure 8: Deviations from the target surface and required average joining forces

4.3. Structural Behaviour

The aim of constructing segmented shells is to interconnect multiple segments to an approximated double-curved structure. Then the governing load-bearing mechanism is membrane action that optimally utilises the complete, but thin cross section. The superposition of stresses from the bending process and the later shell action should be considered.

To compare the structural performance of a single bent strip and a segmented shell, simulations with an incrementally increasing, asymmetric vertical load $q' = \lambda q_0$ were carried out on a surface of revolution of an Elastica curve (Fig. 8).
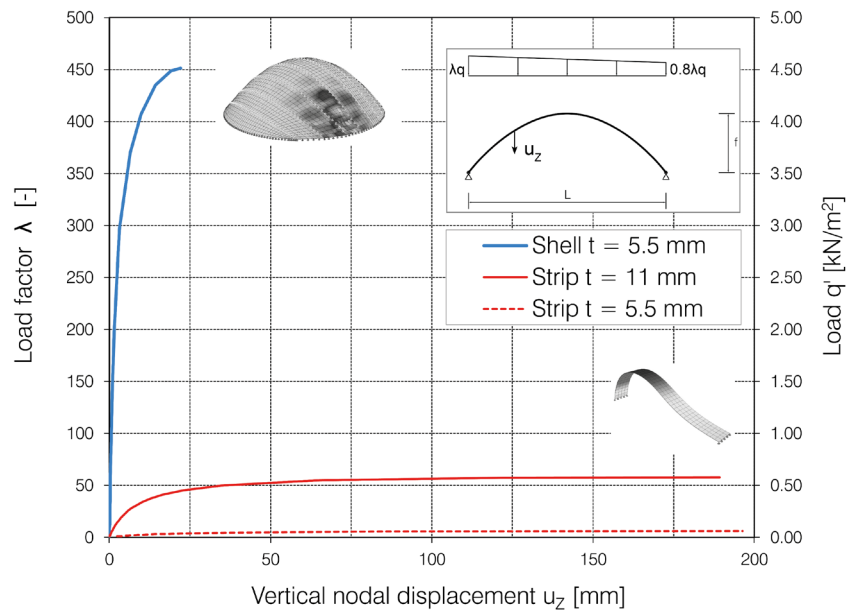

Figure 9: Load-displacement curves for unconnected strips and segmented shell.
Span L = 4.00 m, rise f = 1.20 m, $E_a \sim$ 10000 MPa, specific weight 5.6 kN/m³, q_0 = 0.01 kN/m².

The single strips show large non-linear deformations and snap through buckling as failure mode. Because of the two-dimensional load transfer, the shell instead is stiffer. At a load factor of $\lambda = 200$ a non-linear deformation starts. The critical load factor of $\sim$450 is reached with a maximal deformation of 22 mm under local buckling.

The shell can achieve the efficient load bearing behaviour with a reduced thickness. This causes smaller stresses from the bending process and reduces the amount of necessary material.

4.4 Scale Effect

Because of the large elastic deformations, the cross sections of actively bent members must be thin enough to not exceed allowable stress limits during bending. However, to be used in building construction, they should resist different loads. This brings along the dilemma that common bending-active structures only work in a certain scale range and show stability problems as presented by Lienhard and Knippers [8] or Takahashi *et al*. [14].

To compare the scale-dependent structural behaviour the systems of Section 4.3 are further examined. A factor *s* linearly scales their span, width and height. The thickness is manually chosen in such a way that the structures fail at a given vertical area load $q' = 0.40$ kN/m^2.
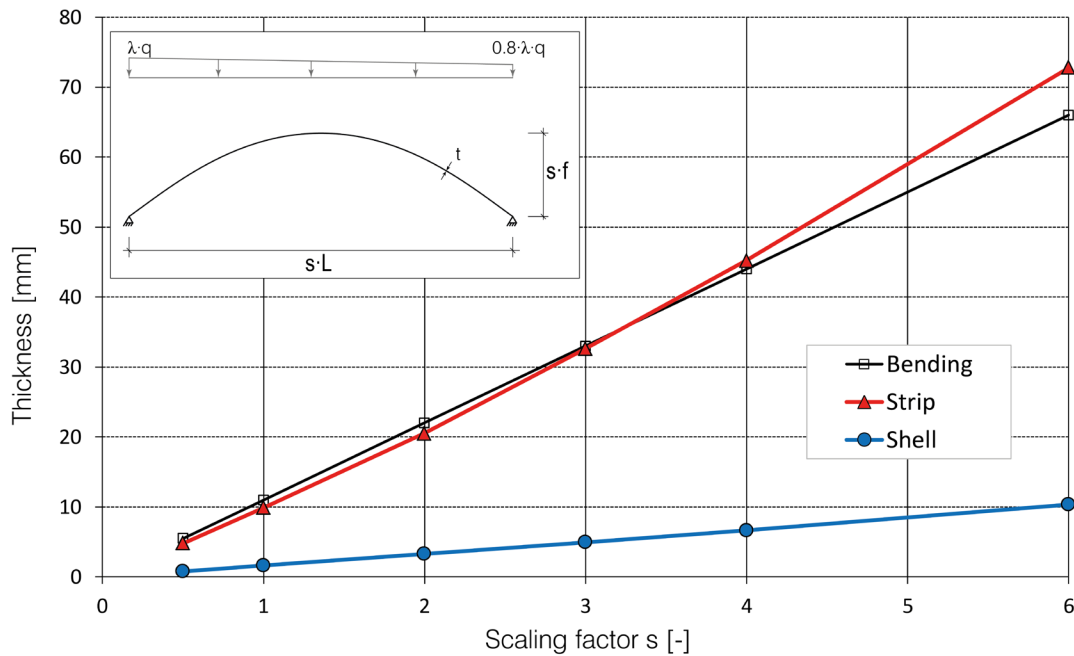

Figure 10: Influence of scale on the possible and necessary thickness of single actively bent strip and shell

The black line in Fig. 10 shows that the thickness of the strip (or shell segment) can be increased linearly with the other system dimensions to reach a consistent stress $f_{m,d}$ by the bending formation. Contrary, the thickness of the single strip has to increase non-linearly to resist the desired load (red curve). From a scale factor of $s = 3.2$ this condition cannot be further satisfied.

The shell instead shows an almost linear behaviour and always lies under the bending curve. Its necessary thickness increases significantly less compared to the single strip. This shows that for appropriate shell forms the coupling of bending-active strips to a segmented shell structure can overcome scaling issues while resisting significantly higher loads than single bending-active elements.

5. Conclusion

This paper introduces the new structural system of Bending-Active Segmented Shells and shows its potential to be used for free-form architecture. The method of adapted bending stiffness achieves complex actively bent shapes and increases the design space of all bending-active structures without the necessity for external shaping-devices. Further, the combination of multiple bent elements to a segmented shell structure allows building efficient structural systems with closed surfaces from planar sheet material through a very simple, moldless construction method. Investigations on the structural behaviour of different shell geometries and under changing scale show the possibility to use the system for buildings.

Further research should address questions of the structural performance of more complex shapes and differing load cases. Additionally, connection details for adjacent strips fulfilling the geometrical and structural constraints need to be investigated. An automated fabrication process for the variable stiffness of actively bent elements would be beneficial to expand the applicability of the system.

Acknowledgements

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